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14. ABSTRACT
An important goal of this proposal is to explore the physics of heteronuclear models: coupled atom-molecule systems where heteronuclear or polar molecules are formed from atoms of distinct species via interspecies Feshbach resonance or photoassociation. The studies conducted to fulfill this goal include the generalization of the technique of the stimulated Raman adiabatic passage to multi-level atom-molecule systems, the detection of exotic phases using electromagnetically induced transparency, and matter-wave bistability and phase separation in the coupled atom-molecule system. Another (extended) goal of this proposal is to explore the physics of cold atom

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Report Title

Coupled atom-polar molecule condensate systems: a theoretical adventure

ABSTRACT

An important goal of this proposal is to explore the physics of heteronuclear models: coupled atom-molecule systems where heteronuclear or polar molecules are formed from atoms of distinct species via interspecies Feshbach resonance or photoassociation. The studies conducted to fulfill this goal include the generalization of the technique of the stimulated Raman adiabatic passage to multi-level atom-molecule systems, the detection of exotic phases using electromagnetically induced transparency, and matter-wave bistability and phase separation in the coupled atom-molecule system. Another (extended) goal of this proposal is to explore the physics of cold-atom mixtures between a (single- or two-component) Fermi gas and a dipolar quantum gas in which the density fluctuation appears in the form of the phonons that obey an anisotropic dispersion spectrum. The studies under this goal include the competition between the triplet superfluid and the singlet superfluid in a 3D dipolar Bose-Fermi mixture, the resonant enhancement of the chiral p-wave superfluid pairings by lowing the energy cost of the phonons the roton minimum in a quasi-2D dipolar Bose-Fermi mixture, and the Cherenkov radiation of Bogolubov phonon modes in a polaronic model in which impurity fermions interact with background bosons in a dipolar condensate.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

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Received		Paper
07/11/2014	12.00	Ben Kain, Hong Y. Ling. Polarons in a dipolar condensate, Physical Review A, (2 2014): 0. doi: 10.1103/PhysRevA.89.023612
07/11/2014	10.00	Lu Zhou, Han Pu, Hong Y. Ling, Keye Zhang, Weiping Zhang. Spin dynamics and domain formation of a spinor Bose-Einstein condensate in an optical cavity, Physical Review A, (6 2010): 0. doi: 10.1103/PhysRevA.81.063641
07/11/2014	11.00	Ben Kain, Hong Y. Ling. Roton-assisted chiral p-wave superfluid in a quasi-two-dimensional dipolar Bose-Fermi quantum-gas mixture, Physical Review A, (9 2013): 0. doi: 10.1103/PhysRevA.88.033616
09/02/2011	1.00	Hong Ling, Michael Fodor. Landau-Ginzburg perspective of finite-temperature phase diagrams of a two-component Fermi-Bose mixture, Physical Review A, (10 2010): 0. doi: 10.1103/PhysRevA.82.043610
09/02/2011	5.00	Hong Ling, Ben Kain. Singlet and triplet superfluid competition in a mixture of two-component Fermi and one-component dipolar Bose gases, Physical Review A, (6 2011): 0. doi: 10.1103/PhysRevA.83.061603
09/02/2011	4.00	Ben Kain, Hong Ling. Vortices in Bose-Einstein condensate dark matter, Physical Review D, (9 2010): 0. doi: 10.1103/PhysRevD.82.064042
09/02/2011	3.00	Lu Zhou, Hong Ling, Han Pu, Weiping Zhang, Keye Zhang. Measurement backaction on the quantum spin-mixing dynamics of a spin-1 Bose-Einstein condensate, Physical Review A, (6 2011): 0. doi: 10.1103/PhysRevA.83.063624
09/02/2013	8.00	Ben Kain, Hong Y Ling. Mixing dipolar condensates: a new opportunity for enhancing superfluid pairing in a spin-polarized Fermi gas, Journal of Physics: Conference Series, (02 2013): 0. doi: 10.1088/1742-6596/414/1/012030
11/02/2012	6.00	Ben Kain, Hong Ling. Superfluid pairing in a mixture of a spin-polarized Fermi gas and a dipolar condensate, Physical Review A, (1 2012): 0. doi: 10.1103/PhysRevA.85.013631
11/02/2012	7.00	Hong Ling, Ben Kain. Cosmological inhomogeneities with Bose-Einstein condensate dark matter, Physical Review D, (1 2012): 0. doi: 10.1103/PhysRevD.85.023527

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Received Paper

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(c) Presentations

1. Hong Ling and Ben Kain, “Polaron properties of a Fermi impurity in a dipolar condensate”, 45th Meeting of the Division of Atomic, Molecular and Optical Physics (DAMOP), Madison, Wiscosin, June 2 – 6, 2014.
2. Hong Ling and Ben Kain, “Nonequilibrium states of a quenched Bose gas”, 45th DAMOP Meeting, Madison, Wiscosin, June 2 – 6, 2014.
3. Hong Ling and Ben Kain, “Polarons in a dipolar condensate”, American Physical Society (APS) March Meeting, Denver, Colorado, March 3 – 7, 2014.

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Received		Paper
07/14/2014	13.00	. Nonequilibrium States of a Quenched Bose Gas, PHYSICAL REVIEW A (03 2014)
09/11/2013	9.00	Ben Kain, Hong Y. Ling. The roton-assisted chiral p-wave superfluid in a quasi-two-dimensional dipolar Bose-Fermi quantum gas mixture, PHYSICAL REVIEW A (04 2013)
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Books

Received	Book

TOTAL:

Received	Book Chapter

TOTAL:

Patents Submitted

Patents Awarded

Awards

Graduate Students

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Hong Y. Ling	1.00	
FTE Equivalent:	**1.00**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED	Discipline
Christopher Rotella	0.00	Physics
Tara Trent	0.00	Physics
FTE Equivalent:	**0.00**	
Total Number:	**2**	

Student Metrics

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Names of other research staff

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FTE Equivalent:
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Inventions (DD882)

Scientific Progress

see the attachment.

Technology Transfer

Contents

I. CREATING GROUND-STATE POLAR MOLECULAR CONDENSATES BY CHAINWISE ATOM-MOLECULE ADIABATIC PASSAGE

A condensate of ground polar molecules with large permanent electric dipoles represents a novel state of matter with long-range and anisotropic dipole-dipole interactions, that are highly amenable to the manipulation by DC and AC microwave fields [1]. As such, creation of such a condensate is expected to be celebrated as another milestone that promises to greatly spur activities at the forefront of physics research, particularly with respect to quantum computing and simulation [2], and precision measurement [3]. The road to molecular condensation is, however, complicated by the fact that more degrees of freedom are needed to describe molecules than atoms. In particular, cooling particles by entropy removal, a direct method popular with atoms, has so far been unable to lower the temperature of molecules down to the regime of quantum degeneracy. Most current experimental efforts in both homonuclear and heteronuclear molecules have, instead, relied on the combination between Feshbach resonance or photoassociation and a single-step stimulated Raman adiabatic passage (STIRAP) to coherently convert atomic condensates into deeply bound molecular condensates. In a single-step STIRAP, there is a relatively large energy difference between the initial Feshbach and final ground molecular states. The former, being close to the dissociation limit, is a highly delocalized state, while the latter is a tightly bound state. It is then, in principle, difficult to locate a single excited state, capable of a large spatial overlap integral [or equivalently a good Franck-Condon (FC) factor] with both the initial and final states. The desire to overcome this obstacle has led to the idea of *chainwise* STIRAP [4], which uses a *single* STIRAP between the initial and final lasers to transfer molecules in

multi-level atomic-molecular condensate systems where additional intermediate states and Raman laser fields are introduced to form a chain of Λ systems.

In this project, we aim to generalize the idea of chainwise stimulated Raman adiabatic passage (STIRAP) [Kuznetsova et al., Phys. Rev. A 78, 021402(R) (2008)] to a chainwise atom-molecule system, in which the role of initial transition is played by photoassociation. Our goal is to create a coherent scheme that allows the two-species atomic Bose-Einstein condensates (BEC) to be directly converted into a ground polar molecular BEC, using a generalized chainwise stimulated Raman adiabatic passage (STIRAP) founded on the concept of atom-molecule dark state, a coherent population trapping (CPT) superposition between stable ground species [5, 6]. Figure 1 is an example of a multi-level atomic-molecular systems. We have formulated a theoretical description of the proposed model within the framework of mean-field field theory; we have developed a perturbation theory which takes advantage that the intermediate lasers are far more stronger than both the initial and final laser fields and have applied the perturbation theory to arrive at a formula which allows the adiabatic condition to be evaluated. In addition to the known advantages, for example, the increased chance to locate pairs of Raman transitions with large FC factors, we have made additional findings. First, atoms are directly converted into ground molecules. Thus, the loss of atoms typically associated with the initial preparation of Feshbach molecules [7, 8] is never an issue here. Second, pulses of longer durations can be employed to meet the adiabatic condition; we can do so because (a) the atom-molecule dark state is far more stable than the molecular dark state, where the initial state is highly unstable compared to the ground (atom or molecule) state, and (b) high power intermediate lasers can be used to combat the loss of molecules from the intermediate states, which can further lengthen the lifetime of the CPT state. Finally, the use of intermediate lasers presents us with new opportunities. In particular, both analytical analysis and numerical simulation indicate that the ratio between adjacent intermediate laser fields, was found to serve as a robust experimental control knob, inaccessible to the usual three-level systems. This control knob together with the stability of the atom-molecule dark state may bring us one step closer to overcome the PA weakness, so that the ground polar molecules can be created directly from degenerate atomic gases in a manner that preserves the phase-space density.

Publications:

Jing Qian, Weiping Zhang, and Hong Y. Ling, "Achieving Ground Polar Molecular Condensates by a Chainwise Atom-Molecule Adiabatic Passage", Phys. Rev. A 81, 013632 (2010).

II. DETECTION OF FERMI PAIRING VIA ELECTROMAGNETICALLY INDUCED TRANSPARENCY

How to distinct and detect different quantum phases in an indisputable fashion has remained a central problem in the study of ultracold atomic physics. Unlike the BEC transition of bosons for which the phase transition is accompanied by an easily detectable drastic change in atomic density profile, the onset of pairing in Fermi gases does not result in measurable changes in fermion density. Here, we propose to take advantage the unique spectroscopic features of electromagnetically induced transparency (EIT), a spectroscopic technique popular with quantum optics community, for detecting the BCS pairing. In our scheme, a relatively strong coupling and a weak probe laser field between the excited state $|e\rangle$ and, respectively, the ground state $|g\rangle$ and the spin up state $|\uparrow\rangle$, form a Λ-type energy diagram, which facilitates the use of the principle of electromagnetically induced transparency (EIT) to determine the nature of pairing in the interacting Fermi gas of two hyperfine spin states: $|\uparrow\rangle$ and $|\downarrow\rangle$ (Fig.

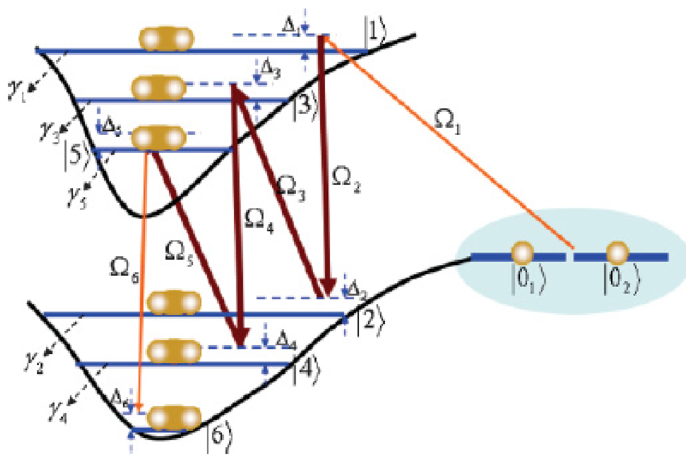

FIG. 1: A schematic of a chainwise STIRAP. A laser field associates atoms from two distinct species of states $|0_1\rangle$ and $|0_2\rangle$ into molecules of state $|1\rangle$ in the excited electronic manifold with a coupling strength Ω_1 proportional to the laser field and the free-bound FC factor. Simultaneously, a series of laser fields of (molecular) Rabi frequency Ω_i $(i \geq 2)$ are applied to move the molecules from the excited to the ground state $|6\rangle$ via additional intermediate energy states.

2). In the EIT method, one can directly measure the absorption spectrum of the probe light. Applying a fast frequency scan to the weak probe field, the whole spectrum can be recorded continuously in an nearly non-destructive fashion to the atomic sample. Furthermore, EIT signal results from quantum interference and is extremely sensitive to the two-photon resonance condition. The width of the EIT transparency window can be controlled by the coupling laser intensity and be made narrower than E_F. Since the EIT method is centered on the transparency window of the probe field, it is considerably less destructive than other optical methods where the spectrum focuses on probe absorption. In addition, due to different selection rules compared with the radio-frequency (RF) method [9, 10], one can pick a different final state whose interaction with the pairing states are negligible, hence avoiding the final state effects.

We have formulated the theoretical description of this problem in two different ways. The first is an approach used more often by people working in the field of quantum optics. The second uses the linear-response theory more familiar to people working in the field of condensed-matter physics. We have introduced a quasiparticle picture and found that in this picture the bare EIT model in Fig. 2 (a) can be compared to a double EIT system shown in Fig. 2 (b). The quasiparticle energy levels consists of a particle (with positive quasiparticle energy) and a hole (with negative quasiparticle energy) branch. The double EIT analogy provides a natural and intuitive interpretation of the origin of the twin spectrum characteristic of the two-species Fermi system when it operates in the boundary between normal Fermi state and BCS phase, namely, each EIT path provides a spectroscopic peak. We have demonstrated that the EIT technique offers an extremely efficient probing method and is capable of detecting the onset of pair formation. With a sufficiently weak probe field and a fast scan of probe frequency, the whole spectrum may be obtained with a nearly non-destructive fashion. A realistic system which avoids the final level effect is shown in 2 (c).

Publications:

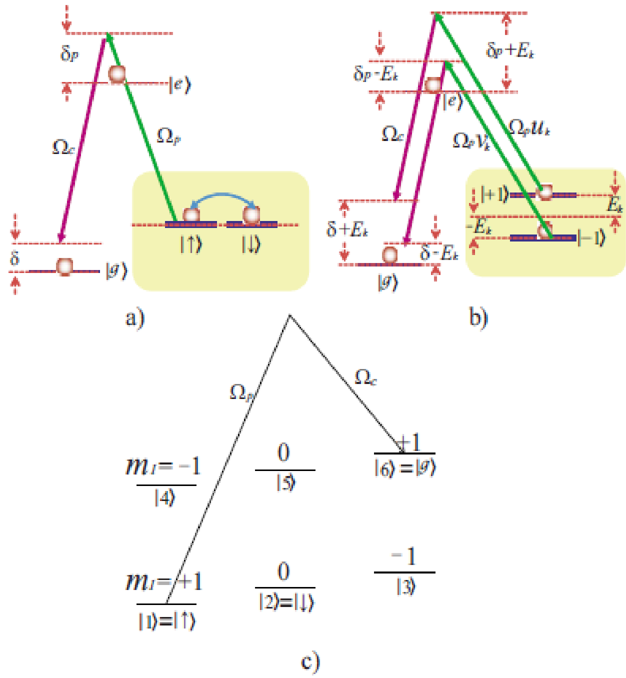

FIG. 2: (a) The bare state picture of our model. (b) The dressed state picture of our model equivalent to (a). (c) A possible realization in ^{6}Li. Here the states labelled by $|i\rangle$ ($i = 1, 2, ..., 6$) are the 6 ground state hyperfine states. Most experiments involving ^{6}Li are performed with a magnetic field strength tuned near a Feshbach resonance at 834G. Under such a magnetic field, the magnetic quantum number for the nuclear spin m_I is, to a very good approximation, a good quantum number. The values of m_I are shown in the level diagrams. Two-photon transition can only occur between states with the same m_I. Any pair of the lower manifold ($|1\rangle$, $|2\rangle$, and $|3\rangle$) can be chosen to form the pairing states. In the example shown here, we choose $|1\rangle = |\uparrow\rangle$, $|2\rangle = |\downarrow\rangle$ and $|6\rangle = |g\rangle$. The excited state $|e\rangle$ (not shown) can be chosen properly as one of the electronic p state.

Lei Jiang, Han Pu, Weiping Zhang, and Hong Y. Ling, "Detection of Fermi Pairing via Electromagnetically Induced Transparency", Phys. Rev. A 80, 033606 (2009).

III. CAVITY-MEDIATED SPINOR CONDENSATE SYSTEMS

The ability of an optical cavity to provide feedback between input and output light fields can result in the modification of the atom-photon interaction in a highly nonlinear fashion. The exploration of such a nonlinear interaction has led to many exciting developments in the study of cavity quantum electrodynamics (QED), including optical bistability [12]. Recent studies in cavity QED focus on cavity systems with a *collection* of ultracold atoms [13–17], motivated mainly by its equivalence [14] to the cavity opto-mechanical system [18], originally proposed as a conceptual model for exploring the boundary between classical and quantum-mechanical systems. As clearly demonstrated in recent experiments [14, 15, 17], such a combination allows us to enter a new regime of cavity QED, where a cavity field at the level of a single photon can significantly affect the collective motion of the atomic

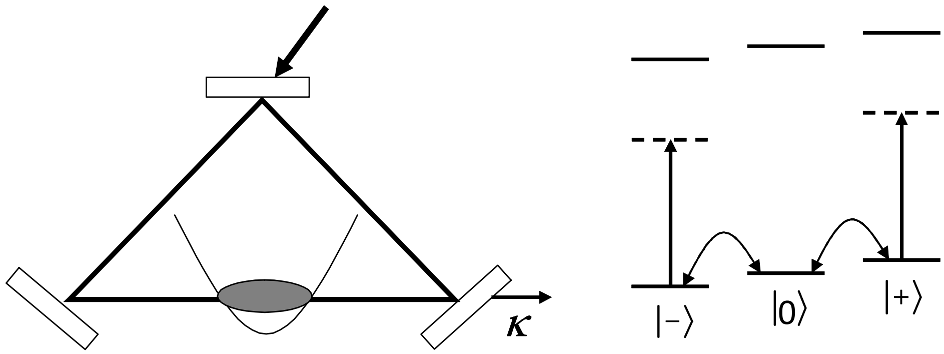

FIG. 3: Schematic diagram showing the system under consideration. We assume that the cavity photons are π polarized which couple the atoms in the $F = 1$ ground-state manifold to the excited manifold with $F' = 1$. The transition selection rule is $\Delta m_F = 0$. However, since the transition $|F = 1, m_F = 0\rangle_g \to |F' = 1, m'_F = 0\rangle_e$ is forbidden, spin-0 level is not coupled by the photon.

samples. This opens new possibilities to manipulate the nonlinear dynamics of ultracold atomic gases with cavity-mediated nonlinear interaction. In this project, we consider a spinor BEC with hyperfine spin $F = 1$ confined in a unidirectional ring cavity (Fig. 3). The cavity operates in the single longitudinal mode and is driven by a coherent pump laser field.

In contrast to the existing works involving in cavity with ultracold atomic gases, which have mainly focused on the interplay between the cavity field and the atomic external degrees of center-of-mass motion [14, 15, 17], we explore the coupling between the cavity field and the internal spin degrees of atomic gases, an intriguing property of a spinor Bose-Einstein condensate (BEC) in which in addition to the repulsive binary collisions, atoms of different spin components can couple to each other via spin-exchange interactions, which give rise to spin mixing, a nonlinear dynamic phenomenon under intense theoretical and experimental investigation. We find that the simultaneous bistability both in optical and matter fields is possible in our model. This result should be contrasted to the current experimental model based on scalar condensates in a Fabry-Perot cavity. There the standing-wave cavity field plays a double role. First, it induces a dynamic phase shift in atoms proportional to the intra-cavity photon number. Second, absorption/emission cycles involving photons propagating in opposite directions provides coupling between motional states of the atom. However, such a coupling is not resonant and cannot be made to be strong in order to avoid spontaneous emission. As a result, the zero-momentum condensate mode always dominates, i.e., the atomic population in finite-momentum modes are rather small. Thus, the matter wave bistability is weak. Our system is the first to exhibit simultaneously both strong bistability in both matter wave and light wave. We also show that the interplay between atomic and cavity fields can greatly enrich both the physics of critical slowing down in spin mixing dynamics and the physics of spin-domain formation in spinor condensates.

Publications:

Lu Zhou, Han Pu, Hong Y. Ling, and Weiping Zhang, “Cavity-Mediated Strong Matter Wave Bistability in a Spin-1 Condensate”, Phys. Rev. Lett. 103, 160403 (2009).

Lu Zhou, Han Pu, Hong Y. Ling, Keye Zhang, and Weiping Zhang, “Spin dynamics and domain formation of a spinor Bose-Einstein condensate in an optical cavity”, Phys. Rev. A 81, 063641 (2010).

IV. ATOM-MOLECULE BISTABILITY WITH AND WITHOUT OPTICAL CAVITY

The ability to cool and trap neutral atoms down to quantum degenerate regime has created a host of new and exciting problems that are increasingly interdisciplinary, bridging in particular the atomic, molecular, and optical physics and the condensed matter physics. The rich knowledge and experience accumulated over the past several decades in these fields have dramatically accelerated the progress of ultracold atomic physics.

In this project, we consider a system where a bosoinc molecule is coupled to two bosonic or fermionic constituent atoms via Feshbach resonance or photoassociation. Such a system not only represents an excellent example that serves to illustrate how the interdisciplinary fields learn and benefit from each other, but also an ideal test ground for studying coupled atom-molecule condensates and the BCS-BEC crossover [19]. The latter is thought to be underlying the mechanism of high temperature superconductors and extensively studied in the realm of condensed matter physics. In addition, the coupled atom-molecule systems have deep quantum optical analogies [20, 21]: bosoinc molecules coupled to bosonic atoms is the matter-wave analog of parametric coupling of photons which has important applications in generating nonclassical light fields and, more recently, in quantum information science; while the system of bosonic molecules coupled to fermionic atoms can be mapped to the Dicke model where a light field interacts with an ensemble of two-level atoms, a model having fundamental importance in the field of quantum optics.

In this project, we have further explored these quantum optical analogies of the atom-molecule system and have focused on the important effects of binary collisional interactions between atoms which are largely ignored in previous studies [20, 21]. We have shown that the atom-atom interaction introduces extra nonlinear terms which, under certain conditions, give rise to matter-wave bistability in both bosonic and fermionic models. Further, we have explored the system inside an optical ring cavity (Fig. 4). The ability of an optical cavity to provide feedback between input and output light fields can result in the modification of atom-photon interaction in a highly nonlinear fashion. We have shown that even when the effective Kerr nonlinearity due to s-wave collisions is not sufficiently negative, bistability can still occur, provided that atom-photon coupling, a key parameter describing the cavity-mediated two-body interaction, is sufficiently large. Hence, we have established the connection between the coupled atom-molecule quantum gases and the nonlinear bistable systems [22] that have been extensively studied in the 80's in the context of nonlinear optics, due both to its fundamental interest, and to its many practical applications in fast optical switches, optical memory, laser pulse shaping, etc.

Papers:

[1] Lei Jiang, Han Pu, Andrew Robertson, Hong Y. Ling, "Matter-wave bistability in coupled atom-molecule quantum gases", Phys. Rev. A 81, 013632 (2010).

[3] Hong Y. Ling, "Bistability in Feshbach resonance ", Journal of Modern Optics, 1362-3044 (2010).

(Note: This proposal was funded one year later after its proposal. I have included some of the progresses I made before it was officially funded.)

V. SINGLET AND TRIPLET SUPERFLUID COMPETITION IN A MIXTURE OF TWO-COMPONENT FERMI AND ONE-COMPONENT DIPOLAR BOSE GASES

The ability to easily mix cold atoms of different species to form new quantum systems brings another exciting dimension to the study of ultracold atomic gases. A single-component Fermi gas only supports

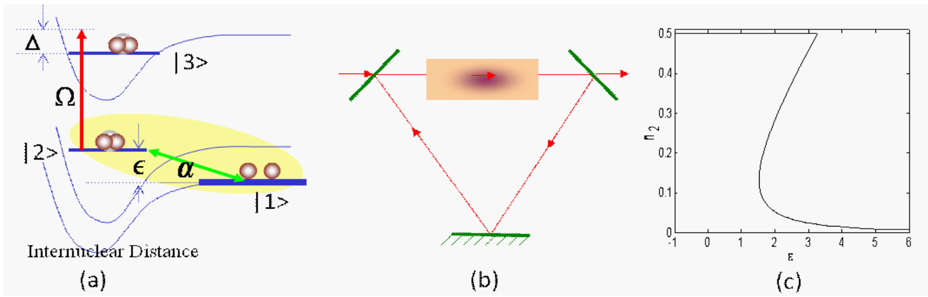

FIG. 4: (a) The energy diagram for a cavity-mediated Feshbach process. (b) A unidirectional ring cavity containing a coupled atom-molecule condensate. (c) The Feshbach molecule density n_2 changes with the Feshbach detuning ϵ in a bistable manner for a special case.

Cooper pairing with odd parities, such as p-wave pairing, which are typically strongly suppressed in accordance with Wigner's threshold law [23]. Mixing bosons induces an attractive interaction between fermions [24], which, as Efremov and Viverit [25] pointed out, raises the prospect of achieving p-wave superfluidity in a Fermi-Bose (FB) mixture. Recently, Dutta and Lewenstein [26] generalized the idea to a two-dimensional (2D) mixture involving dipolar bosons with the goal of realizing a superfluid with $p_x + ip_y$ symmetry whose excitations are non-Abelian anyons that are the building blocks for topological quantum computation [27], and Nishida [28] sought the same goal by mixing fermion gases of different species in different dimensions.

In this project, we consider a three-dimensional (3D) homogeneous mixture between a *two-component* Fermi gas and a *dipolar* Bose gas, made up of two equally populated (balanced) hyperfine spin states of a non-dipolar fermionic atom of mass m_F , and a ground state of a bosonic molecule (or atom) of mass m_B with an induced dipole aligned along the external electric field direction z. The two pseudo spins provide fermions with the opportunity to pair not only via triplet (with odd parities) but also via singlet (with even parities) channels of interaction. This opens up the possibility of using this two-component FB model to emulate and explore pairing physics analogous to that in superfluid ^{3}He [29], which is known to be greatly enriched by the existence of an internal spin degree of freedom.

The induced Fermi-Fermi interaction mediated by a dipolar condensate is also anisotropic in nature and thus opens up a unique avenue for studying superfluids with unusual pairings. The progress in this area has so far been limited, to the best of our knowledge, to a single-component model in a 2D geometric setting [26]. In contrast, the present work expands such studies to a 3D two-component mixture, where both the dipolar interaction between bosons and the s-wave scattering between fermions of opposite spins are independently tunable, and seeks to use it as a model to explore the physics that are currently being hotly pursued in two-component dipolar Fermi gas systems [30, 31]. The recent upsurge of activity in pursuing similar goals but with 3D two-component dipolar Fermi gases [30, 31] has been motivated largely by recent rapid experimental advancement in achieving ultracold dipolar gases both in ^{40}K -^{87}Rb polar molecules [32], and in Cr [33] and spin-1 Rb atoms [34]. Such studies [30, 31] represent a generalization of earlier work [35, 36] aimed at exploiting a $d_{r^2-3z^2}$ type of anisotropy in dipole-dipole interactions for enhancing triplet pairing in single-component dipolar Fermi systems.

In this project, we study in detail the anisotropic nature of the 3D induced interaction, and particularly how one should prepare a two-component FB mixture in order to maximize the opportunity this

induced interaction affords for raising critical temperatures at which phases of different parities begin to compete. We have investigated the optimal conditions for achieving the coexistence between singlet and triplet superfluids in a two-component FB mixture with a dipolar condensate. We have found that the singlet temperature T_s can be made degenerate to the triplet temperature T_t at a temperature seven orders of magnitude higher than $10^{-6}\ nK$, the optimal temperature achievable under a similar set of fixed parameters for a two-component FB mixture with nondipolar bosons. Just as mixing nonlinear waves has been an important means for creating coherent sources of laser light, mixing cold atoms is expected to play an increasingly more important role in creating unique quantum gases (or liquids) in the coming years as the field of ultracold atomic physics continues to mature. The present study reinforces the notion that mixing fermions with dipolar bosons adds another exciting dimension in the pursuit of quantum systems with different and unique properties.

Publications:

Ben Kain and Hong Y. Ling, “Singlet and Triplet Superfluid Competition in a Mixture of Two-Component Fermi and One-Component Dipolar Bose gases”, Phys. Rev. A 83, 061603 (2011).

Presentations:

Ben Kain and Hong Y. Ling, ’Singlet and triplet superfluids in a two-component Fermi - dipolar Bose mixture”, in the 42th Meeting of the Division of Atomic, Molecular and Optical Physics (DAMOP), Atlanta, George, June 13-17, 2011.

“Mixing dipolar condensates: a new opportunity for enhancing superfluid pairing in a spin-polarized Fermi gas”, Department of Physics and Astronomy, State University of New York, Stony Brook, NY, November 19, 2012.

“Superfluid pairing in a mixture of a spin-polarized Fermi gas and a dipolar condensate”, 21th International Laser Physics Workshop, Calgary, Canada, July 23-27, 2012.

“Mixing dipolar condensates: a new opportunity for enhancing superfluid pairing in a spin-polarized Fermi gas”, Department of Physics and Astronomy, University of Delaware, April 24, 2012.

“Mixing dipolar condensates: a new opportunity for enhancing superfluid pairing in a spin-polarized Fermi gas”, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Science, Shanghai, China, March 22, 2012.

“The physics of superfluid pairing in a mixture of a spin-polarized Fermi gas and a dipolar condensate”, Zhejiang University, Hangzhou, China, March 19, 2012.

VI. LANDAU-GINZBURG PERSPECTIVE OF FINITE-TEMPERATURE PHASE DIAGRAMS OF A TWO-COMPONENT FERMI-BOSE MIXTURE

Fermions, constrained by the Pauli exclusion principle, behave very differently from bosons, to which such a principle does not apply. In a many particle system, the former tend to stay away from each other, while, on the contrary, the latter tend to be gregarious. In the most fundamental level, fermions (leptons and quarks) are the building block for all the matter with mass, while bosons serve as the mediator for all the fundamental forces in nature. In spite of this vast difference, when mixed together at temperatures so low that the de Broglie wavelength of the particles becomes comparable to or even longer than the interparticle spacing, fermions and bosons can conspire to create fascinating quantum effects at the macroscopic scale that are of fundamental interest across a broad spectrum of physics,

especially within the disciplines of condensed matter and nuclear physics. As most substances in nature solidify before the temperature could reach the regime where the macroscopic quantum nature of liquid (or gas) can be manifested, the combination of liquid isotopes between ^{3}He and ^{4}He [37] has remained the only laboratory accessible system until recently when the rapid technological advancement in cooling and trapping of neutral atoms has completely turned the situation around. Not only has it resulted in a dramatic proliferation of such systems, including ^{6}Li - ^{7}Li [38, 39], ^{6}Li - ^{23}Na [40], ^{87}Rb -^{40}K[41–44], ^{6}Li - ^{87}Rb [45], but more significantly, with ultracold atomic gases, important parameters, including the interaction between particles of same or different species, can be tuned precisely [43–46], allowing the physics of Fermi-Bose mixture to be investigated in a well controlled manner, in regimes possibly well beyond the reach by traditional solid state systems.

Recently, by changing the Feshbach detuning across a certain critical point at which all the minority atoms pair up with the majority ones to form a molecular Bose condensate, the group at MIT [47] has successfully created, from a two-component Fermi mixture with population imbalance, a quantum gas where Bose molecules are mixed with spin-polarized (unpaired majority) fermions. In addition to confirming an earlier theoretical prediction of the existence of a transition from full miscibility to phase separation [48, 49], using the same system, they were able to determine the effective dimer-fermion scattering length within a reasonable agreement with the prediction made more than 50 years ago [50] but never verified experimentally, once again demonstrating that the ultracold atom system provides an excellent experimental platform for testing theories.

Inspired by this work, instead of one Fermi state as in a single-component Fermi-Bose mixture in Ref. [47], we consider a two-component Fermi-Bose mixture involving a hyperfine state $|b\rangle$ of a bosonic atom with mass m_B and two equally populated hyperfine states: spin up $|\uparrow\rangle$ and spin down $|\downarrow\rangle$ of a fermionic atom with mass m_F. The latter Fermi system when equipped with Feshbach resonance has been the main source of inspiration for much recent excitement in the forefront of ultracold atomic physics, due chiefly to the vital role it plays in the study of crossover from Bose-Einstein condensation (BEC) of tightly bound atom pairs to nonlocal Bardeen-Cooper-Schrieffer (BCS) atom pairs. Thus, mixtures of bosons with such Fermi systems shall be widely accessible to experiments.

A unique advantage of a cold atom system is that its system parameters can be precisely tuned, allowing it to access a larger regime of phase diagrams. Further, availability of detection techniques, such as absorption laser imaging of densities and radio-frequency (RF) spectroscopy [9], makes the experimental determination of such phase diagrams in fine detail possible. In contrast to the existing works for the two-component model [24, 51], which have various purposes, we focus on producing phase diagrams in the space made up of chemical potentials, which are intensive statistical variables that must remain invariant among the separated phases and hence uniquely define a phase separation [52]. This is to be contrasted to spaces where coordinates are served, for example, by particle number densities, of which separated phases in a phase separation have different values [51]. We have performed a systematic study of the finite-temperature phase diagram in the chemical potential space for a two-component Fermi-Bose mixture with attractive Fermi-Fermi and repulsive Fermi-Bose interaction. Using a combination of scaling and Landau-Ginzburg theory, we have identified, within the framework of mean-field theory, a set of features generic to the phase diagrams for such mixtures. Further, we have applied the theory to explore the physics of pairing among fermions in a tightly confined trap surrounded by a large BEC gas.

Publications:

Michael Fodor and Hong Y. Ling, “Landau-Ginzburg perspective of finite-temperature phase diagrams

of a two-component Fermi-Bose mixture", Phys. Rev. A 82, 043610 (2010).

VII. COLLECTIVE EXCITATIONS AND STABILITY OF A SUPERSOLID PHASE.

The ability of symmetry breaking to generate different phases has been what makes the world we live colorful and interesting. One of the most actively sought phases both in theory and in experiment has been supersolid, a macroscopic quantum phase that displays both superfluid and crystalline order, a fascinating property originating from the simultaneous breaking of two symmetries of completely different nature: a continuous global $U(1)$ gauge symmetry and a discrete translational displacement symmetry. The recent rapid development in cold atomic physics has greatly inspired ideas and proposals aimed at exploring ultracold atomic systems as alternative platforms to realize supersolid, a subject which has traditionally belonged primarily to the realm of solid state physics, where the experimental efforts focused mainly on solid ^{4}He [53].

The interest in the formation of supersolid in continuous (as oppose to lattice) ultracold atom models dates back to 2004 when Santos et al.[54] discovered that owing to the long-range and partially attractive nature of the dipolar interaction, the collective excitation of a dipolar Bose-Einsten condensate (BEC) exhibits, at finite momentum, an analogous "roton" minimum observed in the excitation spectrum of superfluid helium. This combined with the ability to precisely tune key parameters, such as interaction strength and trap shape, has greatly raised the prospect of achieving supersolid phase in a dipolar BEC. However, the ability to drive the roton minimum across zero to the region where the collective excitation becomes imaginary only signals the instability of the uniform dipolar condensate- it represents a precursor of a transition to a density wave, but it does not automatically mean that the transition from the uniform superfluid to the stable superfluid density wave (supersolid) phase [55][56] can be materialized in practice.

As matter of fact, a subsequent numerical investigation in the quais - two-dimensional (2D) trap setting by Komineas and Cooper [57] neither yielded stable nor produced metastable supersolid phase in the parameter region close to roton instability. This prompted Dutta et al. [58] to modify the atom-atom interaction and they claimed that in a Bose-Fermi mixture, the Femi induced boson-boson interaction can stabilize the density wave. A stable density wave was also shown to be possible in a 1D setting by Giovanazzi et al. [56] using a trick to flip the sign of a dipole-dipole interaction. Henkel et al. [59] traced the inability to locate the stable density wave in the quasi-2D dipolar BEC to the same partially attractive dipole-dipole interaction (in real position space) that generates the 2D roton excitations; they, instead, sought to realize stable regular density waves using an isotropic van der Waals interaction potential with a softened core that is entirely repulsive in real position space but exhibits a window of negative potential in the momentum space.

The central question surrounding all these efforts is how one should design an atom-atom interaction potential that is capable of producing a stable supersolid phase. In the solid state system, the ability to control the two-body interaction is quite limited. In contrast, important parameters, particularly, the interaction between two atoms can be engineered in the ultracold atomic physics, for example, by subjecting the dipolar interaction to both AC and DC electromagnetic fields [61], by modifying the boson-boson interaction by mixing bosons with fermions [58], etc. A phase study is not complete without carrying out a linear stability analysis against small fluctuations around the interested phase [60]. In this project, we consider a one-dimensional bosonic quantum gas where the momentum

dependent two-body interaction characterized with a roton minimum can give rise to the supersolid phase. We aim to derive, from a simple model with three momentum modes, features that are generic to the excitation spectrum of the supersolid phase in a realistic model where far more than three modes are used. We find that the spectrum is marked with two sound modes. We also find that the spectrum at the zone edge is prone to becoming imaginary at a sufficiently large interaction and the instability first sets in at the edge. We plan to generalize this study to two-dimensional settings and use it as one of useful tools for engineering the two-body interactions that can give rise to stable supersolid phases.

Presentations:

Blake Laing and Hong Y. Ling, 'Density waves of a dipolar condensate in a two-component degenerate Fermi gas", in the 42th Meeting of the Division of Atomic, Molecular and Optical Physics (DAMOP), Atlanta, George, June 13-17, 2011.

VIII. CHIRAL p-WAVE SUPERFLUIDS IN DIPOLAR BOSE-FERMI QUANTUM GAS MIXTURE

The chiral p-wave ($p_x \pm ip_y$) phase in both superconductors and ultracold atomic systems has attracted significant attention in recent years mainly because its vortex core supports a unique bound quasiparticle (non-Abelian) excitation with zero energy known as a Majorana fermion. A two-dimensional (2D) topological system with well isolated vortices can be thought of as a quantum computing device where a unitary transformation (braiding vortices [64]), the basic operation of any quantum computer, is performed in a degenerate space topologically protected from other nontopological excited states by an energy gap [64]. The advent of cold-atom systems opened up the possibility of using electromagnetic fields to tune and design two-body interactions capable of enhancing chiral p-wave superfluid pairings. The same goal may also be achieved in a Bose-Fermi mixture since bosons can induce and hence modify the Fermi-Fermi interaction in a Fermi gas. In a dipolar quantum gas [65, 66], the dipole-dipole interaction represents a control knob inaccessible to nondipolar bosons (Fig. 5). Thus, mixing dipolar bosons with fermions opens up new possibilities [26, 67, 68]. Experimentally, great progress has been made in recent years in achieving dipolar quantum gases consisting of either heteronuclear molecules with electric dipoles [69] or atoms with magnetic dipoles [33, 34].

In this project, we consider a quasi-two-dimensional (2D) cold-atom mixture of nondipolar fermions in a spin-polarized Fermi gas and bosons in a dipolar condensate where all the dipoles are oriented along the axial direction. The collective excitation of such a system was discovered [54] to exhibit, at finite momentum, a minimum analogous to the "roton" minimum in superfluid helium. It was shown recently that the existence of dipolar bosons in such a model can significantly enhance the unconventional superfluids in the spin-polarized Fermi gas [26]. The underlying physical mechanism [26] is the ability to enhance the induced Fermi-Fermi attraction by lowering the energy cost of the phonons of the dipolar condensate near the roton minimum [54]. In this project, we investigate how the enhanced induced interaction, usually ignored in the stability analysis of the Bose-Fermi mixture, affects the stability property of the mixture, and how the same enhanced interaction affects the effective mass of the fermions near the Fermi energy and hence the p-wave superfluid critical temperature.

We have studied, within the Hartree-Fock-Bogoliubov approach, the p-wave superfluid pairings in a quasi-2D dipolar Bose-Fermi mixture. In a quasi-2D trap setting, the competition between the

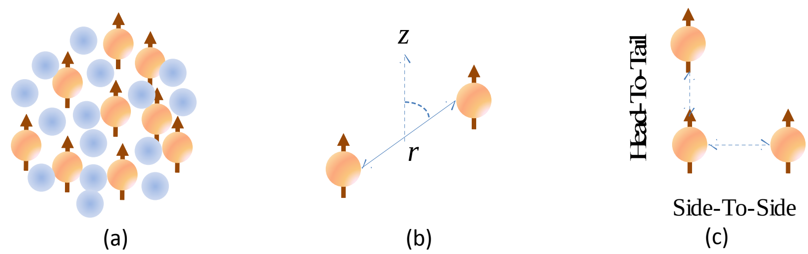

FIG. 5: (Color online) (a) A mixture between a spin-polarized Fermi gas of nondipolar fermions (blue spheres) and a condensate of dipolar bosons (orange spheres with arrows). (b) The dipole-dipole interaction between two dipolar bosons separated by a distance r. (c) The side-to-side (head-to-tail) arrangement where the dipole-dipole interaction is the most repulsive (attractive).

attractive and repulsive part of the dipole-dipole interaction can lead to the roton structure in the phonon spectrum of the dipolar condensate. Mixing dipolar bosons with fermions in quasi-2D space offers the opportunity to use the roton minimum as a tool for engineering the phonon-induced attractive interaction between fermions. The present work demonstrates that enhancing the induced interaction by lowering the roton minimum can affect the stability properties of the mixture as well as the effective mass of the fermions in an important way. It also shows that one can tune the system to operate in stable regions where chiral p-wave superfluid pairings can be resonantly enhanced by lowing the energy cost of the phonons near the roton minimum.

Publications:

Ben Kain and Hong Y. Ling, "Mixing dipolar condensates: a new opportunity for enhancing superfluid pairing in a spin-polarized Fermi gas", Journal of Physics: Conference Series, 414 (1), 012030(1-8).

Ben Kain and Hong Y. Ling, "The roton-assisted chiral p-wave superfluid in a quasi-two-dimensional dipolar Bose-Fermi quantum gas mixture", Phys. Rev. A 88, 033616.

Presentations:

Ben Kain and Hong Ling, “P-wave superfluid in a quasi-two-dimensional dipolar Bose-Fermi quantum gas mixture”, APS March Meeting, Baltimore, Maryland, March 18-22, 2013.

IX. IMPURITY FERMIONS IN A DIPOLAR CONDENSATE

A conduction electron in an ionic crystal or a polar semiconductor displaces nearby ions, thereby polarizing the medium in the vicinity of the electron. An analogous picture emerges when an impurity atom is immersed in an ultracold atomic quantum gas containing atoms different from but capable of interacting with the impurity atom. In recent years, much efforts have been focused on systems where both impurity and background atoms are fermions, inspired by the remarkable agreement between theoretical predictions [70, 71] and experimental findings obtained from set-ups where a single spin-$\downarrow$ impurity is immersed in a sea of spin-$\uparrow$ background atoms [72, 73], a setting that shares much resemblance to that of Kondo problem in condensed matter physics. Recent years have also witnessed an increased interest in systems where impurity atoms are fermions but background atoms are bosons [large (continuous) polarons and small (Holstein) polarons] due largely to their parallel to the electron-phonon system where the polaron picture is central to the understanding of colossal magnetoresistance

FIG. 6: (Color online) (a) Fermionic impurities immersed in the sea of a dipolar condensate. (b) A schematic of Bragg scattering, in which two off-resonance laser beams with wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ and frequencies ω_1 and ω_2, tuned far closer to the impurity atomic than the background atomic transitions, are incident upon the cold gas cloud, and an impurity atom can undergo a two-photon stimulated scattering, which changes its momentum by $\hbar\mathbf{k} = \hbar\mathbf{k}_1 - \hbar\mathbf{k}_2$ and its energy by $\hbar\omega = \hbar\omega_1 - \hbar\omega_2$.

materials and is also believed to play a vital role in the physics of high-T_c superconductivity in strongly correlated materials and in unconventional pairing mechanisms.

In this project, motivated by recent experimental advancement in achieving dipolar quantum gases consisting of either heteronuclear molecules with electric dipoles [69] or atoms with magnetic dipoles (see, for example, references [33, 34]), we consider the same polaronic models except that the background atoms are now bosons in a dipolar condensate (Fig. 6). In a dipolar quantum gas, the dipole-dipole interaction represents a control knob inaccessible to nondipolar bosons. Thus, mixing dipolar bosons with fermions opens up new possibilities. An important consequence of the dipole-dipole interaction is that the phonon spectrum of a dipolar condensate is no longer isotropic - one can tune the dipolar interaction to lower the energy of a phonon along some directions while simultaneously increase it along other directions (a phenomenon that a recent experiment aims to demonstrate using Raman-Bragg spectroscopy in a dipolar chromiumin BEC [74]). Thus, impurity fermions submerged in a dipolar condensate act as anisotropic polarons owing to their interaction with the surrounding phonons. For anisotropic crystals in solid state systems, electron-phonon interaction is anisotropic, but since the electron-phonon coupling is typically weak, the anisotropic effect on the polaron physics remains typically small. The goal of this project is to explore strongly anisotropic polarons in Fermi-dipolar Bose mixtures.

We have considered a polaronic model in which impurity fermions interact with background bosons in a dipolar condensate. The polaron in this model emerges as an impurity dressed with a cloud of phonons of the dipolar condensate, which, due to the competition between the attractive and repulsive part of the dipole-dipole interaction, obey an anisotropic dispersion spectrum, with the energy being lower along the radial direction than along the axial direction. By capitalizing this anisotropy, we have shown that the ability to tune the dipolar interaction, s-wave scattering lengths, and the mass ratio between the impurity and background atoms make this model an excellent platform to study strongly anisotropic polarons. We have studied how the anisotropy affect the spectral function of the impurity fermions, which is directly accessible to the momentum resolved rf spectroscopy experiments. We have also discussed how the anisotropy affect the Čerenkov radiation of Bogoliubov phonon modes, which can be directly verified by experiments by letting the dipolar BEC moving against an obstacle in the form of the localized optical potential of a far-detuned laser beam.

Publications:

Ben Kain and Hong Y. Ling, "Polarons in a dipolar condensate", Phys. Rev. A 89, 023612 (2014).

Presentations:

Hong Ling and Ben Kain, "Polaron properties of a Fermi impurity in a dipolar condensate", 45th Meeting of the Division of Atomic, Molecular and Optical Physics (DAMOP), Madison, Wisconsin, June 2 – 6, 2014.

Hong Ling and Ben Kain, "Polarons in a dipolar condensate", American Physical Society (APS) March Meeting, Denver, Colorado, March 3 – 7, 2014.

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